From the tall tower to a lush garden

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ABSTRACT

The cumulative nature of mathematics is often cited as being responsible for compounding a sense of failure and resultant disaffection with mathematics at school. The *tall and spindly structure* of school mathematics curriculum makes it a hard climb for most students. Considering that this structure dates back to the days of the industrial revolution, it is worth revisiting the structure in light of the information revolution. We suggest a different metaphor, that of a garden, to invoke alternate structuring of curriculum that may offer opportunities for revisiting core concepts and techniques in many different contexts. The focus is on providing key mathematical insights rather than competence in mathematics aimed towards a career in science or engineering.

KEYWORDS
Curriculum, curricular structure, reasoning, computational thinking.

1. Importance and reluctance

In 1996, I participated in an exercise conducted by the Tamil Nadu Science Forum (TNSF), a voluntary group: a survey was conducted among the general public to ascertain their views on science and mathematics education. The survey was carried out by volunteers in urban and rural areas alike, in 8 locations in different districts of Tamil Nadu. Regarding mathematics, the first question posed was: “Do you think that mathematics is important, should it be compulsory at school?” This was followed by: “Did you enjoy learning mathematics at school?” Then the last: “Given another opportunity, would you like to learn mathematics again?” Similar questions were asked about science as well. The survey was informal, meeting a cross section of society, with nearly a thousand individuals responding, almost all of it conducted in public spots like bus stands. The study was meant only for internal use by TNSF, to improve its science communication efforts.

The results were interesting. There was almost a consensus of affirmation in answer to the first question. Mathematics was pereived to be very important, and everyone was certain that it should be taught as a compulsory subject at school. Several people volunteered a reason for it as well, that mathematics was very useful in life. When it came to the second question, there was considerable unease. A large majority said that they had disliked mathematics at school. Many volunteered that they regretted not

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having learnt mathematics "properly." When it came to the third question, the reaction was invariably one of horror, though there were a few enthusiastic souls wanting another opportunity to learn mathematics.

Methodologically, this survey was not scientific, and from such informal questioning one cannot draw conclusions on how the general public engages with school mathematics. For TNSF, enthusiastically engaged in working with science and mathematics education in schools, there were problems of interpretation. The group was committed to enhancing community involvement in elementary schools and the survey was supposed to help us shape community participation. But here was a puzzle to solve. How can a society consider something so important that its children should be compelled to learn it while at the same time not want to directly engage with it? If it were truly important, why would they not want it for themselves as well? Is it an expression of parental ambition, that something that the parents had dreaded should be accessible easily to children?

It was also interesting to note that most people who talked of doing badly in mathematics at school blamed only *themselves* for it, and not the school, or the teachers, or the textbooks. Psychologists should explain this phenomenon of self-certification, but one has to wonder at the system that achieves this: the system itself emerges blameless, all failure self-attributed to individuals. In fact, there is considerable literature the world over, on what perceived failure in mathematics does to the self-esteem of teenagers GN (2018). The observation that at any point in time, a considerable part of humanity is being subjected to significant psychological stress in the process of learning mathematics, is sobering, especially in light of a social consensus that such learning is important. In fact, we can call this a burden that mathematics education must address.

The *Learning Without Burden* report (LWB 1993) identified **the burden of non-comprehension** resulting from rote learning to be the central load on children in schools. In the case of mathematics learning, the burden is heavier, since importance is also attached to success in mathematics: inevitably, if one must succeed whether one comprehends or not, one is pushed towards rote learning, thus adding to the burden.

2. The shape of mathematics curriculum

Mathematics has for long been associated both with the world of work and the world of abstraction. If measurement and commerce, architecture and agriculture, encompassed mathematical practice, astronomy and aesthetics were built on mathematical abstraction. At both ends, mathematical ability was recognized and appreciated, though always limited to a few. In Tamil Nadu, the *payol schools* that existed before the British era taught Babu (2012) the arithmetic of grain (large numbers) and that of gold (miniscule fractions). Committing tables to memory and performing feats of computation mentally was important.

In Europe, the industrial revolution raised a major demand for mathematics and science education. Rather than the mathematics of the ancients, the more utilitarian and experimental applications of mathematics was demanded by industry. By mid-nineteenth century, mathematics learning had helped to engrain a class division that remains endemic to mathematics education: mathematics for the workers and mathematics for the 'well educated' Noyes (2007). By 1870s the modern subject-oriented curriculum emerged, but the needs of military and technical schools which were flourishing by then, shaped school curricula in science and mathematics. By the early twentieth century, mathematics as a discipline was addressing foundational anomalies and redefining itself rigorously in an abstract axiomatic manner. Put somewhat crudely, this resulted in a tension between two orientations, one towards the polytechnic and the other towards the university. In England, the former 'won' the battle, and this was the influence the newly created system of mass education in India was subjected to. Calculus was the pinnacle of this system, and all mathematical learning inexorably led to it.

This twenty-second movie of the history of mathematics education inevitably presents an oversimplified trajectory. The colonial encounter was more complex, but it essentially led to the abandonment of a mathematical discipline that rested on the two pillars of work and astronomy and replaced it with a tower of abstraction. Interestingly, the justification for the new structure was again in its being useful: but the notion of usefulness moved away from the experienced everyday life of the citizen to the working life of the skilled worker in an industrial society. Science and technology were creating new materials, new industries and new modes of production, and if there was a single mathematical tool was powerfully employed in all these industries, it was calculus. Indeed, mathematical modelling came to mean setting up of a differential equation; invariably, it was difficult to solve, until the computer came along and offered numerical solutions.

Thus emerged the multi-layered tall and spindly shape of school mathematics. Arithmetic forms the foundation in primary school, then simple algebra in middle school, then geometry, then more algebra, then trigonometry, some more geometry making use of all of this, and then right on top of it all, the epitome of all this mathematical knowledge: calculus. One could call it a layered cake, if one finds it tasty. For most students, it is a narrow dreary tower to climb, dark and with few windows to see the attractive scenery outside. There might be breathtaking beauty awaiting those who make the climb, but for many the effort is way too much.

This structure has many consequences:

(1) Each course of study is a primarily a prerequisite for the next one, without having an intrinsic interest of its own. For children, such delayed gratification, of it being useful in some distant future, is meaningless.
(2) If one misses out in aquiring competence at one level, then the next (for which this is a prerequisite) becomes very difficult to negotiate, slowing one down, resulting in inadequate competence at that level, thus compounding the problem one level even higher. Many simply give up.
(3) This effectively prevents informal development of intuition. Since procedural fluency at one level is needed to negotiate concepts at the next level, attaining this becomes urgent and takes over as the main activity. But then, if one has learnt procedure but has no intuition, then at the next stage, procedural learning is easier than conceptual understanding and the story is repeated. Thus students who do emerge on top are often masters of set procedures rather than having a rich toolkit of techniques.
(4) Much of the development is to create tools, that once obtained, make the preceding development irrelevant. Once one can set up equations and solve, several arithmetical tools learnt earlier are no longer needed. Once you learn matrices, solving equations also becomes routine. And so on. Mathematics teaches you to kick the ladder you climbed on, you no longer need it. But this also contributes to the sense of unfamiliar landscape each level brings you to.

The Position Paper of the National Focus Group on the Teaching of Mathematics (NFG 2005) refers to the 'cumulative nature of mathematics' as one of the reasons that cause fear of mathematics in school: "If you struggle with decimals, then you will struggle with percentages; if you struggle with percentages, then you will struggle with algebra and other mathematics subjects as well." There are very few opportunities to restart on these at a later stage and quickly catch up, though this might actually be possible for many. This has indeed to do with the tower shaped structure of the curriculum, whereby one climbs up rung by rung. An opportunity to revisit and relearn requires a structure that allows multiple entry points to subject matter. The Position Paper refers to this: "the shape of mathematics education has become taller and more spindly, rather than broad and rounded."

It is to be noted that other subjects of study at school do not possess such a cumulative tower-like structure. The natural sciences offer multiple points of entry, allowing for change of interests in the child over the early teen period. With physical and life sciences offering a diversity of approaches, with social studies raising a range of questions in the child, and language study offering almost endless revisits and opportunities to acquire skills missed out earlier, the tower of mathematics stands alone, visible from everywhere, singular and forbidding for those outside on the ground.

On the other hand, a different image is as relevant. The roots of mathematics are deeply embedded in human experience and form an intricate and vast network under the soil. As each mathematical tree shoots up, layers of abstraction create branches that put out leaves, but there is continuity and connectedness from root to leaf, and perhaps this is what curricula should emphasize, and provide educational experiences of these connections.

A clarification is (badly) needed here. Many mathematics textbooks introduce a topic with the observation that this particular concept is important, is extensively used in "real life applications". This is merely intimidation, with an implied threat that you are missing out on something important if you do not master the concept. Such existence of "applications" is not the rootedness we are speaking of. When the abstraction is seen to arise from the child's own lived experience it attains authenticity. This could come from a game, an activity, or even from the realm of fantasy (dear to children), or even an experience that a child does not have but can relate to.

Rootedness is an obligation to present the origin of the abstraction; if appreciating the origins calls for maturity that the child may not yet have, either postponement or provision of an opportunity to revisit the topic later may be called for.

The *Learning Without Burden* report (LWB 1993) argues that the "distance between the child's everyday life and the content of the textbook further accentuates the transformation of knowledge into a load." In the case of mathematics, where abstraction from experience is critical as a prelude to playing with abstraction in itself, the continuity referred to above becomes important. This cannot be achieved by giving primacy to abstractions and merely illustrating them with "examples from daily life". As an aside we should nod to yet another issue raised by the (LWB 1993) report: these 'examples from everyday life' rarely include the life of a farmer or a labourer, reinforcing the impression that mathematics stands apart from the lives of ordinary people, that mathematicians are not ordinary people.

On another note, this structure reinforces the social perception of school mathematics as a pipeline for human resources that flows from schools to scientific careers. Viewed thus, the perception we referred to earlier makes sense: scientific careers are intellectually on 'top', requiring the hard climb 'up'; if one cannot, it is one's incapability. The tower invokes respect and awe.

3. Is curriculum the problem?

The foregoing discussion raises the question: is there a way to reshape, reimagine mathematics curriculum, that yields a different structure, one that allows multiples entries, exits and revisits not only to curricular themes but also for the process of aquiring skills and fluency?

3.1. *Curriculum vs pedagogy*

Before we address this question, it is important to consider the reform process initiated by the National Curriculum Framework 2005 (NCF from now on, (NCF 2005)). It was principally set in the paradigm of constructivist pedagogy, and called for a "shifting of focus from content to process." This shift in pedagogy addressed some of the major concerns raised in the *Learning Without Burden* report. The principle of pedagogy preceding curriculum can be seen to dominate the narrative in the *Mathemagic* series, the new textbooks for the elementary stage that the NCF resulted in. To a significant extent, this has indeed influenced classroom experiences in primary schools. However, when we consider the textbooks and classroom experiences at the upper primary or secondary stage, it is clear that disciplinary curriculum takes over to precede pedagogy and by the time we reach the higher secondary stage, there are few explicitly pedagogic considerations to point to. In itself, this need not be a failing, and may only attest to an unreasonable burden the pedagogic expectations from the NCF may have placed on the curriculum.

Indeed, it is a matter of primacy of concern. When high school life is viewed merely as acquiring entry qualification for tertiary education, it lacks importance in itself and the system does not commit resources to the tremendous overhaul needed for pedagogic transformation. The needs of those who exit the system after high school carry little weight in the system either. Thus, during high school, evaluation precedes assessment which precedes curriculum which overrides pedagogy, resulting in yet another tower structure, against which the recommendations in (NFG 2005) seem ineffective.

All this is to point out that without rethinking curriculum, it is hard to address the concerns of universalization and the pedagogic transformation envisioned by (NCF 2005) effectively. The curricular acceleration process and the 'tall and spindly' curricular structure of school mathematics, acknowledged but unaddressed by the NCF reform process, render a process focus in the classroom very difficult. On the other hand, a healthy predisposition towards problem solving, and mathematical practices such as making conjectures, looking for counterexamples, drawing analogies, generalization when specifics are hard, devising multiple representations and so on, advocated by (NFG 2005) are sufficiently important for us to interrogate the intransigence of the curriculum that allows little room for such processes.

3.2. *Procedure vs concept*

Emphasis on procedure at the cost of conceptual understanding is often blamed for the burden of incomprehension in mathematics at school. Indeed, when memorizing multiplication tables and learning arithmetical operations as algorithms to be performed takes precedence in primary school, this does happen. When solving linear and quadratic equations is learnt as procedure, this occurs in middle school. By the time a student enters high school, such enactment of procedure has become an expectation from the student, and the habits persist, whether it has to do with construction in geometry, finding standard deviation for given data, inverting a matrix, or integration by parts. All these admit a clear procedure to be employed and problem solving is reduced to recognizing which algorithm is to be employed, to deconstruct the problem statement and pick data that would form the input to the algorithm, and run the set procedure on the given data. The procedure implements a relation, say $R(x_1, \ldots, x_n)$ and learning amounts to recognizing which of the arguments are given and which are to be computed.

Indeed, procedure without meaning works against the spirit of mathematics. But we must not absolve the curriculum of all responsibility for such undue emphasis on procedure and treat it only as a pedagogical issue. The structure of the curriculum demands that procedures be learnt in intricate detail at each stage, with meaning or not. This observation is important enough to itemize:

- The structure of the curriculum allows for the possibility of moving up the ladder all through school without clarifying concepts at each stage.
- The structure of the curriculum requires that procedual competence be ensured at each stage to navigate the next stage easily.

There are two ways to negate this: the stringent would attack the first and insist on ensuring conceptual clarity at each stage, thus preserving the primacy of the tower structure. What we suggest is the other option, to allow for some laxity in procedural competence at a stage, allowing for multiple revisits and opportunities over many stages to acquire it.

The conclusion is inescapable. If we are to emphasize mathematical thinking, the curriculum needs to be examined accordingly. Whether it needs change or not, at the least, it requires restructuring.

Here, a quote from William Thurston (Thurston 1990) is appropriate:

> The long-range objectives of mathematics education would be better served if the tall shape of mathematics were de-emphasized, by moving away from a standard sequence to a more diversified curriculum with more topics that start closer to the ground. There have been some trends in this direction, such as courses in finite mathematics and in probability, but there is room for much more.

4. Reshaping curriculum

So much has been written about mathematics curriculum in school that it seems unlikely that anyone can ever say anything new. However, every curricular reform follows its own path, every generation seeks to reshape curriculum in its own manner. For instance, the history of mathematics curricula in school is often presented as a seesawing conflcit between procedural emphasis and conceptual emphasis, with the pendulum swinging one way and another (Sriraman 2012).

Our account is almost exclusively from an Indian viewpoint, considering the changes in school mathematics since the Learning Without Burden report. This is already a vast terrain since every state has its own Board of Education and the changes are neither homogeneous across the country nor are synchronized. As a rule, the states tend to follow the lead from the centre in carrying out reform but the nature of the reform itself may be different in a state (Ramanujam 2012). For instance, Tamil Nadu followed up the Yash Pal Committee with its own Sivagnanam Committee in 1997 (of which this author was a member), but the recommendations of the latter were very different in spirit. Similarly, after NCF (NCF 2005), many states defined their own curricular framework, with significant departures. In fact, while the Centre has not offered another comparable curriculum document since 2005, Tamil Nadu published a framework document in 2006 and another one in 2017.

However, in the context of mathematics education, the differences have been minor, most of which have related to syllabus structure, rather than on curricular or pedagogic themes. The emphasis on mathematisation and processes in NCF 2005 has had an impact on state curricula as well, but typically they also follow (NCF 2005) and critiquing curricular structure and leaving it well alone. In pedagogy, a significant point of departure is the linking of school textbooks iin Kerala with dynamic geometry software, thereby rendering greater flexibility to content transaction. Perhaps the only significant *curricular* shift is the recent one in Tamil Nadu, presented in the 2017 document and relaised in textbooks during the years 2018 and 2019. This is the introduction of an Information Processing track in mathematics curriculum for the elementary stage, subsuming the earlier data handling and statistics units, and introducing components of computational thinking. The recent textbooks for Classes 1 to 8 have implemented this curricular shift.

From the perspective of the last two decades, and viewing the next two decades (in a haze), one feature stands out in Indian reality: the impact of the mobile phone and the information revolution. The great transformation of production, manufacture and distribution processes due to computing has not caused a deep impact on Indian society as yet, but it may do so in the decades ahead. But the transformation of everyday life due to the communication and information technologies is still ongoing and it is conceivable that algorithms will be running the lives of Indians across the social spectrum in the coming decades. This observation raises many issues of relevance to all education, but in the context of mathematics, we note some simple propositions:

- The rootedness of mathematics in the physical world and the world of experiences is even more important to emphasize, as the virtual world gains in importance in the lives of children.
- Calculational needs stemming back to the industrial revolution and deeply embedded in extant mathematics curricula can be done away with in favour of tools. However, gaining control over the tools requires the ability to *verify answers provided by tools*, and this is an intrinsically mathematical ability. To give a crude example, answering 170×26 could be left to an 'app' on the phone, but a sanity check that the answer is between 3500 and 5000 is more important, as mistakes in typing could give wild answers.
- From learning procedures, mathematics needs to move to *reasoning about procedures.* Assessment has mainly required students to enact the procedures they have learnt. A shift to critiquing procedure, whereby the student gets to compare different procedures for the same task and adjudge them, consider variations of the procedure for different purposes, and even design procedures on their own requires a new language of discourse in mathematics. This is essential as students grow up in a world run by algorithms: a critical outlook calls for competence in evaluating algorithms (and not in enacting them).
- Mathematics textbooks provide a great deal of information (though perhaps to a lesser extent than other subjects of study), and the student is expected to commit these to memory. In the new century, with quick access to information, this can be done away with, whereas the emphasis needs to shift to *evaluating information.*

- The search for patterns has always been central to the study and practice of mathematics. While we perceive with the eye and understand with the mind, the mind dominates the eye in mathematics. But mathematicians and technologists find new ways to see patterns now, both with the eye and with the mind. The term *visualisation* has acquired tremendous depth in recent times, and this ability can change mathematics education as fundamentally as digital computing has changed engineering practice.
- School mathematics is deeply committed to providing the language of science and engineering (thanks to the historical trajectory discussed earlier), but in the last 50 years, mathematics has greatly expanded its reach: into economics, finance and banking, business practice, medicine, the social sciences and more. This is principally due to recognition of order and pattern of many kinds, not only those of number and shape. Then the *language of mathematics* taught at school should reflect this breadth.
- The gravest issues faced by humanity in the twenty-first century, call for an understanding of global patterns like climate change, scarcity of natural resources and the spread of pandemics. For mathematics education, this implies an engagement with *models and uncertainty* all through. Again, the potential offered by new technologies for this engagement is to be utilised.

These considerations, along with the provision of educational opportunities that allow for revisiting concepts and skills at many stages, once again make the case for a curricular shape that is broad and allowing lateral navigation. While working out such a detailed curriculum is beyond the scope of this essay and well beyond the competence of the author, certain elements can be identified.

There are many big ideas of mathematics but the foregoing discussion highlights a few essential themes: *quantity, shape, dimension, transformation, uncertainty.* These can be viewed through patterns and activity that result in a language of discourse gradually gaining in precision and abstraction. In a sense, the desired outcome should be seen as fluency in this language rather than specific knowledge or skills. As with all language use, different speakers would be skilled in different idioms and registers, a few becoming writers and creators.

4.1. *The mathematics of* things

Mathematician and educator W W Sawyer wrote: *Do things, make things, notice things, arrange things, and* only then *reason about things.* This is sound advice for mathematics education, especially at the elementary stage, as we want children to engage with the physical world. (Again, this is of increasing importance in a world where the virtual makes inroads into children's lives.) Hands-on mathematics gives opportunities not only for learning number, shape and quantity, but also for computational thinking through experiences in counting, arranging, organizing.

Working with things in the elementary stage offers extensive opportunities for engaging with quantity, shape and measurement, It also offers the scope for reasoning about procedures, and evaluation of tools. When we ask for distributing 45 toffees equally among 15 children we perform a procedure (whether carried out or not). But then when you ask for verification that all toffees have been counted and all children have been counted and that everyone has equal number, we reason about the procedure. When we can find different ways of counting out (one toffee to each child until the toffees are exhausted; or divide the class into two groups, make two heaps of toffees and distribute one pile among each; or if the children are already seated in 4 rows, make 4 piles and distribute) there is an extension of the idea. When we argue that they all accomplish the same task, the reasoning goes further. When we can also consider which of these procedures is better suited for a particular circumstance we are laying the foundation for critiquing algorithms.

Of course, a single task with small children should never be overloaded in this manner. This was merely an attempt to point out that working with things can accomodate reasoning and verification of the kind mentioned earlier. Moreover, giving precedence to *making* things gives children early experiences of models, of transformation and change, of measurement and dimension, and to a lesser extent, of uncertainty.

Perhaps the most important aspect of according primacy to things, is that skill in arithmetical operations can be entirely relativised to the needs arising from the work context. Viewed thus, long division and multiplication of three-digit numbers would get less importance than working with equivalence of fractions. While competency in operations can then be seen as desirable, what is essential and insisted on by evaluation mechanisms would be their use in context.

4.2. *Measurement*

Measurement of length, area and volume are important in school, and arise from everyday experience. However, the curricular space that measurement occupies is small, reduced to the learning of some formulas. Processes such as estimation and approximation, and concepts such as accuracy, precision and error are generally glossed. In fact experience with measurement of geometric quantities is only one aspect of this learning. Measurement of arithmetic quantities such as size, count, order and variations in labelling is important and needs to be integrated into the child's thinking as well. Measurement in the context of random variation is enjoyable from an early age: counting how many times a number comes up in repeated throwing of dice, or in spinning discs, or in other data is not only interesting; lack of pattern is important for perception. Measurement of change or variation is an exciting challenge from early on. Experiencing the complexity of measurement can perhaps help to build numeracy in the child: she may grow up to be less accepting of claims arising from the misuse of data and statistics.

4.3. *Transformation*

Symmetry and transformation turn up everywhere in mathematics, and this is perhaps the deepest link between what makes it most useful as well as makes it a tool of aesthetics. Experiencing invariances under flips and rotations, counting symmetries and looking for transformations that cause a breaking of some symmetry can be experienced from an early age. Culturally rooted mathematical practices provide an engaging opportunity in this regard, as for instance in the case of kolams in Tamil Nadu. Looking for symmetry in parts, as in the growth of natural objects from repetitive patterns of molecules or cells offers room for mathematical reasoning and computational thinking in its grasp of iterative processes. The use of technological tools and programming environments can greatly enhance these experiences that can be built on in the secondary school.

That repetition can be the source of accuracy, symmetry, or chaos is a deep insight of mathematics, and we can provide curricular opportunities at various stages to understand iteration and repetition.

4.4. *Functional variation*

Observing and quantifying change is central to science, and mathematics provides the language for such use in school. However, the algebraic and geometric contexts that develop this language remain disparate, as also topics like 'ratio and proportion' relegated to commercial mathematics. The notion of linearity, central to scientific study, is hardly accorded centrality.

Change comes in many forms: linear, periodic, continuous, smooth, random, and so on. The realisation that they can all be articulated (with ease or with difficulty) in the language of mathematics is valuable in many domains of life. What is needed is exposure to these forms. Tools can greatly help in this realisation, though mastery in the idiom can well be left to later stages of specialisation in the university.

4.5. *Abstractions*

Interestingly, while abstraction in mathematics is often blamed for its difficulty, the fact that mathematics studies abstractions actually never figures prominently in mathematics at school. The use of symbols is prevalent but inventing symbolic representations is never a conscious activity. The notion of identity, equivalence, similarity and change are central to all mathematics but they are never engaged with at school. Recursion pops up in the definition of the factorial function, but is mostly never seen as a powerful technique of definition. Logic is relegated to a curricular unit termed mathematical reasoning, rather than seen to pervade the practice of mathematics as it does.

Asking the question why a mathematical procedure (such as solving quadratic equations) is reliably correct, or why an equation is solvable in one number system and not in another are essentially logical questions. These are not questions to be asked in children's annual examinations, but ignoring them altogether deprives children of the opportunity to appreciate how abstraction functions in mathematics.

4.6. *Uncertainty*

Appreciating regularity and understanding uncertainty are closely related. This is a profound insight that mathematics can provide, and one of immense use not only in intellectual endeavour but also in democratic life. Building models and understanding models is possible only with this realisation. This again calls for continued engagement through school life at various stages, using mathematical resources available at that stage.

5. Discussion

We began with the recounting of a survey in which people were asked, "Given another opportunity, would you like to learn mathematics again?" As it turned out, a couple of years later, we undertook a mini-course for adults, most of whom had been to high school, some had higher education as well, and wanted this opportunity. This was not part of any certificate programme in adult education, and the curriculum was freewheeling. The course was structured on themes such as Quantity, Shape, Measurement, Transformation, Optimization, Variation and Uncertainty. It was aimed at offering exposure to mathematics, rather than providing competency in a specific area.

In science education certain propositions areconsidered central to scientific literacy, for instance, that all matter is made up of atoms. There are certain deep insights in mathematics that are no less central, for instance, that dimensionality is a property of space but also a means of ordering knowledge. Insights are not gained by learning specific items of knowledge or by acquiring specific skills, but by gradually gaining strength of conviction over a long period of time, through immersion in educational activity.

A curriculum that seeks to provide such insights would present mathematics not as a forbidding tower of great height but as a garden of lush variety: with plants flashing flowers of dazzling colour, and tall fruit bearing trees under whose shade children are at play.

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